Crease Enhancement Diffusion

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Ridge and valley structures are important image features, specially in oriented textures. Usually, the extraction of these structures requires a pre-filtering step to regularize the source image. In this paper, we show that classical diffusion based filters are not appropriate for this task, and propose a new filtering process. This new filter can be tuned to join broken valley or ridge lines while preserving their junctions, thus preserving the general topology of the lines obtained afterwards. This filter can be interpreted as an example of the intrinsic introduction of structure on diffusion processes. **Keywords**: diffusion process, structure tensor, ridges, valleys, anisotropic diffusion, PDE's.

1. INTRODUCTION

Ridge and valley structures are important linear image features, specially for highly oriented textures. Usually, the extraction of these structures requires a pr-filtering step to regularize the original image. There are many works devoted to the extraction of ridge/valley structures [3, 8, 9]. In any case, some kind of regularization of the input image is needed to obtain stable and meaningful results. Frequently, this regularization consists simply of a convolution by a Gaussian kernel. In case of highly oriented textures, this approach is unsuitable because the inner structures can be destroyed. A solution is to use non isotropic filters, like the proposed by Perona and Malik [10, 11]. However, it has proved to be more appropriated the use of an explicit directed diffusion process. A recent approach in this context has been provided by Weickert [12, 13, 14]. In his work, he proposes a new diffusion process based on the structure tensor, which provides a local description of the anisotropy in a neighborhood of the image. This new diffusion process has two important properties: it makes possible to close some interrupted linear structures and enhances reliable linear structures in the input image. On

the other hand, an undesirable effect of this diffusion process is that ridge/valley junctions are destroyed and non linear structures are deformed. In this paper we present a new diffusion process that enhances ridge/valley structures. This process is based on a new diffusion tensor that includes the differential structure of ridges and valleys in an image. This tensor, that we call *creaseness tensor*, provides the creaseness direction in each point and a local measure of creaseness. In Sect. 2 we review what is a diffusion process. In Sect. 3 we define the creaseness tensor and construct the corresponding diffusion process. Section 4 presents comparative results. Finally, Sect. 5 summarizes the main conclusions.

2. DIFFUSION PROCESSES

Diffusion processes derive from Fick's law [5] and the continuity equation. Fick's law expresses that a gradient concentration leads to a flow which compensates it. If we include Fick's law into the continuity equation, which expresses that mass is only transported but can neither be created nor destroyed, we obtain the diffusion equation:

$$\frac{\partial u}{\partial t} = div(D \cdot \nabla u) \tag{1}$$

where D is a diffusion tensor and u corresponds to the mass concentration. The diffusion tensor define the diffusion process, for example, if we choose D as the identity matrix we have a particular case of the diffusion equation, the heat equation. This special case can analytically be solved and one can prove that it corresponds to a convolution of the initial function u with a Gaussian kernel. If we think of u as the grey level of an image I, we obtain the Gaussian smoothing as a special case of the diffusion equation [7, 15]. This diffusion process suffers from the well known problem of displacement of edges due to Gaussian blurring. In order to solve this problem, Perona and Malik proposed another diffusion process [10] (see also [11]) which can be seen as a special case of the diffusion equation:

$$\frac{\partial I}{\partial t} = div\left(\frac{1}{1 + \frac{\|\nabla I\|^2}{K^2}} \cdot \nabla I\right) \tag{2}$$

where K is a parameter controlling the diffusion strength. However, neither the Gaussian diffusion nor the Perona-Malik's are tuned to enhance linear structures while regularizing the image. In these processes the diffusion directions are always collinear to the image gradient ∇I and its perpendicular $\nabla I^{\perp}$. Then, with the purpose of enhancing linear structures, Weickert proposed in [14] the tuning of the diffusion directions according to the dominant orientation at each image pixel. This dominant orientation at each pixel is obtained through the structure tensor analysis [6]. The structure tensor can be defined as the convolution of the Gaussian kernel with the tensor product of the regularized gradient by itself:

$$S_{\rho,\sigma} = G_{\rho} * (\nabla I_{\sigma} \nabla I_{\sigma}^T) \tag{3}$$

The parameter σ is the *differentiation scale* and controls the size of the objects whose orientation has to be determined, while ρ is the *integration scale* and controls the size of the neighborhood in which an orientation is dominant. Weickert

proposed a diffusion process based on the construction of a diffusion tensor whose eigendirections coincide with the eigenvectors of the structure tensor, but having different eigenvalues, namely:

$$\begin{cases} \lambda_1 = \alpha \\ \lambda_2 = \alpha + (1-\alpha)\exp^{\left(\frac{-C}{(\mu_1-\mu_2)^{2m}}\right)} \end{cases} \tag{4}$$

where μ_1 and μ_2 are the eigenvalues of the structure tensor and m, C and α are parameters controlling the exponential shape and the diffusion strength in the non coherence direction. Since this diffusion process enhances coherent flow-like structures it seems appropriate for regularizing the image before extracting ridges and valleys. However, we have observed that the inner structures of ridges and valleys are modified by this filtering process. Specially, many junctions disappear and original structures are deformed. In order to try to solve these problems we propose a new diffusion process that introduces information about the *crease structure* of the image.

3. CREASE DIFFUSION FLOW

In this section we propose a new diffusion process that smoothes the original image while preserving and enhancing the intrinsic crease structure. First, we make a brief review of the classical theory surrounding the problem of surface structure classification, giving an intuitive and geometric interpretation of it. Then we make use of this classical theory in order to define a new diffusion tensor, that we call *crease diffusion tensor*.

3.1. Local surface classification

Consider a gray level image $I(x,y)$ as a surface graph. Then we can describe this image locally by means of its Taylor expansion. In fact, if we are only interested in the shape of the image we only need the second order derivatives information, that is the Hessian of I. The Hessian provides us an analytical local representation of a surface $z = u(x,y)$ in each point $p = (x_0, y_0)$. The corresponding eigenvectors and eigenvalues, that we denote as v_1, v_2, h_1 and h_2 respectively, can be used to make the classification presented in Fig. 1 (left) in terms of surface local structure. In figure 1 (left) we can see the local surface classification obtained using only the Hessian information. But there exists a more accurate definition for ridges and valleys using also the gradient information. That is, we consider a point to be a ridge/valley point if the gradient is orthogonal to the eigenvector corresponding to the greatest eigenvalue of the Hessian. In addition, the following relationship between the surface principal curvatures (k_i) and Hessian eigenvalues (h_i) holds:

$$k_i = \frac{h_i}{\sqrt{1+\|\nabla u\|^2}} \quad if \ \ \nabla u \cdot v_i = 0 \tag{5}$$

That is, in the case of ridges and valleys the eigendirection v_i and the *normalized eigenvalue* (k_i) coincide exactly with the principal direction and the maximum principal curvature respectively. We can obtain directly this normalized eigenvalues for the Hessian if we analyze the *normalized Hessian* below instead of the classical
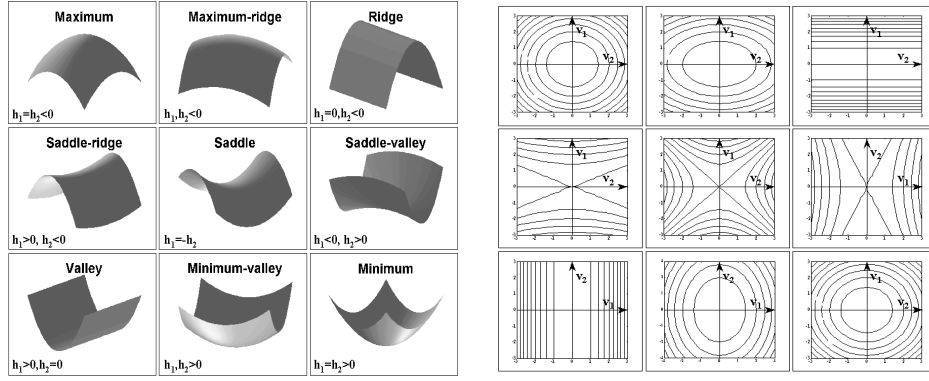
FIG. 1. *Left: Surfaces obtained varying the h_1 and h_2 eigenvalues of a quadratic function. Right: Level sets corresponding to the surfaces on the left and its eigendirections corresponding to $\bar{k}_1$ and $\bar{k}_2$ respectively.*

one,

$$\frac{1}{\sqrt{1+\|\nabla u\|^2}} \begin{pmatrix} \frac{\partial^2 u}{\partial x^2} & \frac{\partial^2 u}{\partial x \partial y} \\ \frac{\partial^2 u}{\partial x \partial y} & \frac{\partial^2 u}{\partial y^2} \end{pmatrix} \tag{6}$$

Unfortunately, for our purposes the classification provided in Fig. 1 (left) is not adequate for two main reasons. The first reason is that we are looking for a positive valued descriptor which reaches its highest value in the presence of ridges or valleys. On the other hand we also need a bounded descriptor. In order to achieve these requirements we propose the following shape descriptor:

$$\mu = \frac{\tilde{k}_1 - \tilde{k}_2}{\tilde{k}_1 + \tilde{k}_2} \tag{7}$$

where $\tilde{k}_1 = \max(|k_1|, |k_2|)$ and $\tilde{k}_2 = \min(|k_1|, |k_2|)$. It is easy to prove that μ is bounded, concretely $0 \leq \mu \leq 1$. Since we are interested in ridges and valleys separately we can consider the following factorization of the descriptor μ:

$$\mu_r = \begin{cases} \frac{\tilde{k}_1 - \tilde{k}_2}{\tilde{k}_1 + \tilde{k}_2} & if \quad k_1 < 0 \\ 0 & if \quad k_1 \geq 0 \end{cases} \qquad \mu_v = \begin{cases} \frac{\tilde{k}_1 - \tilde{k}_2}{\tilde{k}_1 + \tilde{k}_2} & if \quad k_1 > 0 \\ 0 & if \quad k_1 \leq 0 \end{cases} \tag{8}$$

where μ_r and μ_v reaches its highest values in the presence of ridges and valleys respectively. In fact we obtain the classification of the Table 1 (note that $\mu = \mu_r + \mu_v$). The direction associated with the largest eigenvalue (in absolute value) corresponds to the local *crease direction* of the surface. In Figure 1 (right) we can see the crease direction (labeled as v_2).

3.2. The Creaseness Diffusion Tensor

In order to construct a multilocal representation of the *normalized Hessian* (6) we can make a double regularization step as in the case of the structure tensor (see

TABLE 1

Different surfaces grouped in function of $\tilde{k}_1$, $\tilde{k}_2$.

$\tilde{k}_1 > 0$ $\tilde{k}_2 = 0$	Ridge Valley	$\mu = 1$
$\tilde{k}_1 > \tilde{k}_2$	Saddle-Valley Saddle-Ridge Maximum-Ridge Minimum-Valley	$0 < \mu < 1$
$\tilde{k}_1 = \tilde{k}_2$	Minimum Maximum Perfect Saddle	$\mu = 0$

eq. (9)). First we regularize the initial image u in order to obtain robust derivatives (derivation step) and then regularize the tensor field (integration step):

$$G_\rho * \left[\frac{1}{\sqrt{1+\|\nabla u_\sigma\|^2}} \begin{pmatrix} \frac{\partial^2 u_\sigma}{\partial x^2} & \frac{\partial^2 u_\sigma}{\partial x \partial y} \\ \frac{\partial^2 u_\sigma}{\partial x \partial y} & \frac{\partial^2 u_\sigma}{\partial y^2} \end{pmatrix} \right] \tag{9}$$

where G_ρ is a Gaussian kernel of size ρ. This tensor provides a multilocal version of the eigendirections and normalized eigenvalues of the Hessian. We denote as v_1 the first eigendirection corresponding to the highest eigenvalue in absolute value ($\tilde{k}_1$) and v_2 the one corresponding to the lowest. The *creaseness diffusion tensor* is constructed in order to have the same eigenvectors v_1 and v_2 and the following associated eigenvalues:

$$\begin{cases} \lambda_1 = \epsilon \quad \epsilon \in (0,1) \\ \lambda_2 = \alpha\mu_r + \beta\mu_v \quad \alpha, \beta \in [0,1] \end{cases} \tag{10}$$

where α and β are parameters controlling the diffusion strength in the presence of ridges and valleys respectively and ϵ controls the diffusion in the direction perpendicular to the creaseness direction and assures the semidefinite property of the diffusion tensor. Due to the regularization steps, the diffusion process which we propose here is well posed and the existence and uniqueness of solutions can be proved in the same way as in [2, 4]. Interesting values for the couple (α, β) are $(1, 0)$, pure ridge diffusion, $(0, 1)$ pure valley diffusion and $(1, 1)$ ridge-valley diffusion.

4. RESULTS

Figure 2 shows the behavior of our filter both in a synthetic image and on a real fingerprint image. The synthetic image (Fig. 2 left)is formed by valley (darker) and ridge (brighter) structures. In the central part the valleys and ridge are more radial while as we move away from the center the crease structure become more like concentric ellipses. The original image (top left) was filtered using our proposal (left bottom) and their valleys were extracted (right hand of each image). We can observe that the filtering process (which is tuned to enhance mainly the valley structures) enhances the valleys of the original image leading to a better results on the valley extraction. We have performed the same experiment in the case of the fingerprint. One can also notice that the valley structure was enhanced leading to a
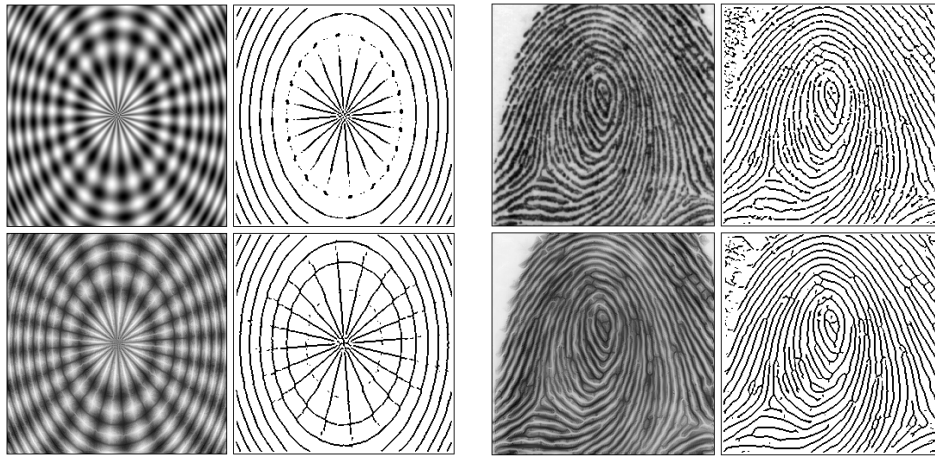
FIG. 2. *Left: Parameters $\alpha = 1$ $\beta = 0.5$ $\sigma = 1$ $\rho = 0.5$. Right: Parameters $\alpha = 1$ $\beta = 1$ $\sigma = 1$ $\rho = 0.5$.*
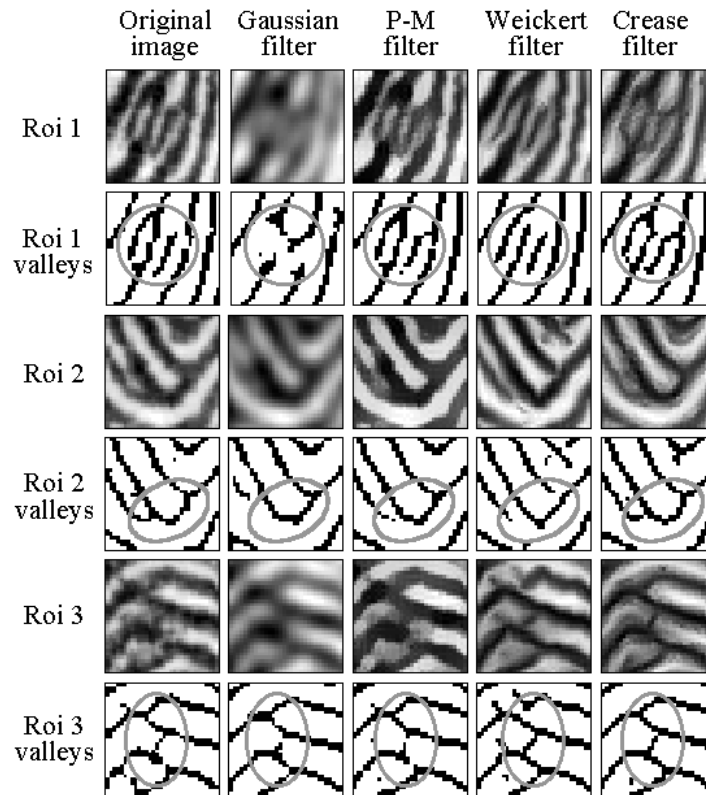

FIG. 3. *ROIs from the fingerprint image (Fig. 2 right side). Each ROI was filtered using Gaussian smoothing (second column, $\sigma = 2$), Perona-Malik's smoothing (third column, $K = 4$, 10 iterations), Weickert's smoothing (fourth column, $\sigma = 1, \rho = 1$, 10 iterations) and finally our filter (fifth column, $\sigma = 1, \rho = 1, \alpha = 1, \beta = 0.6$, 10 iterations).*

better valley extraction. The fingerprint is analyzed more precisely in the following figure.

In Fig. 3 we zoom three regions of interest (ROIs of 32×32 pixels) from the fingerprint image (Fig. 2 right). Each ROI was filtered using Gaussian smoothing (second column), Perona-Malik's smoothing (third column), Weickert's smoothing (fourth column) and finally our filter (fifth column). The corresponding valleys (row below each ROI) were also computed in order to compare the effects of the filtering step. The junctions in each valley image are surrounded with a gray oval in order to make easy their localization. Although the structures are more preserved in the case of the Perona-Malik smoothing than in the case of Gaussian smoothing we can observe that some linear structures and also some junctions are destroyed. Weickert's filter preserves much more the linear information but also the topology (junctions) are destroyed. In our case we observe a better behavior. Linear structures are enhanced while preserving its own topology.

5. DISCUSSION

We have presented a new diffusion process to enhance ridge/valley structures, that we term crease enhancement diffusion. It is based on two main ideas: a directed diffusion process and a stable bounded creaseness operator. The advantages of this filtering process are that while crease features are enhanced, artifacts in the image do not appear, contours are preserved and junctions are not interrupted. We have compared it with other classical filters like Gaussian or Perona-Malik smoothing, showing their problems, specially in the case of highly oriented textures. Also we have compared it with a recent approach proposed by Weickert, the coherence enhancement diffusion, from which we have borrow the idea of coherence. We have shown that although Weickert's filter enhances linear structures (like ridges and valleys) many junctions disappear and some artifacts appear, that is, the crease structure is not preserved. Our filtering process avoids these problems to a large extent. This new filter should be also interpreted as an example of introducing the enhancement of specific geometric structures in diffusion processes. This is done directly in the diffusion process, and not explicitly favored via outliers as in [1].

ACKNOWLEDGMENT

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