Error Analysis for Lucas-Kanade Based Schemes

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Abstract. Optical flow is a valuable tool for motion analysis in medical imaging sequences. A reliable application requires determining the accuracy of the computed optical flow. This is a main challenge given the absence of ground truth in medical sequences. This paper presents an error analysis of Lucas-Kanade schemes in terms of intrinsic design errors and numerical stability of the algorithm. Our analysis provides a confidence measure that is naturally correlated to the accuracy of the flow field. Our experiments show the higher predictive value of our confidence measure compared to existing measures.

Keywords: Optical flow, Confidence measure, Lucas-Kanade, Cardiac Magnetic Resonance.

1 Introduction

The dynamics (motion and deformation) of the myocardium reflect, in a higher or lower level, most of the cardiovascular diseases. In order to explore the properties of motion across image pixels, the computation of a dense flow field is mandatory. Variational schemes are widespread powerful tools for computing dense motion vectors. In the last years there has been an increasing interest in developing variational schemes for minimizing over-regularization and keeping motion discontinuities [1, 2]. Therefore advanced techniques are able to detect irregular discontinuities of motion at injured myocardiums. However, their application to decision making in medical imaging requires discarding those regions where optical flow (OF) is neither reliable nor accurate.

The most common way of measuring OF accuracy is by computing its deviation from the true motion vector. This suffices to quantify the overall performance, but it is useless at locating areas of poor performance in real-life sequences where no ground-truth is given. Existing confidence measures rely on either local image structure, the computed flow and statistical patterns. Measures based on local image structure assess flow fields by taking into account some features of the input image like the determinant, the gradient or the structure tensor [3]. Confidence measures that only consider the computed flow field,

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such as the local inverse of the energy functional evaluated over the computed flow field [4] are linked to the computation method. Finally, statistical confidence measures [5, 6] are based on the estimation of the flow distribution from a training data-set. It follows that they are independent of the particular variational formulation or numerical scheme. A main limitation for their application to medical imaging is that they require a huge database including any pathological pattern. Given the unpredictable nature of pathological cases, this is not feasible. A main concern is that none of the above confidence measures have been defined taking into account the error sources of the numerical schemes.

This paper presents an error analysis of Lucas-Kanade schemes [7, 8], which achieve successful results in a wide range of medical applications [9–11]. We consider that an algorithm has two main sources of error: a deficient design of the formulation and stability of the numerical scheme. In this context, this paper contributes in the design of a confidence measure in two aspects. Analysis of the error sources of OF schemes that require solving a linear system [7, 8] and the introduction of a confidence measure based on the numerical stability of the algorithms. Comparison to existing measures using the Middlebury database show a higher correlation between our measure and flow end point error. Experiments on Cardiac Magnetic Resonance (CMR) sequences illustrate its potential in medical applications for detecting regions with a non-reliable OF.

2 Error Analysis of Lucas-Kanade Based Schemes

The Lucas-Kanade (LK) approach [7] is based on the assumption that OF keeps constant in a neighborhood of each pixel of size σ. Under this assumption the OF $w = (u, v)$ solves:

$$\underbrace{\begin{pmatrix} K_\sigma * (I_x^2) & K_\sigma * (I_x I_y) \\ K_\sigma * (I_x I_y) & K_\sigma * (I_y^2) \end{pmatrix}}_{A_{LK}} \begin{pmatrix} u \\ v \end{pmatrix} = \underbrace{\begin{pmatrix} -K_\sigma * (I_x I_t) \\ -K_\sigma * (I_y I_t) \end{pmatrix}}_{b_{LK}} \tag{1}$$

for $I(x, y, t)$ a sequence image, subscripts partial derivatives (x and y for spatial derivatives and t for temporal ones), $*$ the convolution operator and K_σ a Gaussian kernel of standard deviation σ.

The LK approach can be formulated in global terms [8] using the following variational framework

$$E_{LKV}(u, v) = \int \psi_1(E_{LK}) + \alpha \psi_2(|\nabla w|^2)\, \mathrm{d}x\, \mathrm{d}y \tag{2}$$

for $E_{LK} = \mathrm{w} K_\rho * (\nabla I \nabla I^\top) \mathrm{w}^\top$, $\mathrm{w} = (u, v, 1)$, $\psi_i(s^2) = 2\beta_i^2 \sqrt{1 + \frac{s^2}{\beta_i^2}}$ and β_i a scaling parameter. The associated Euler-Lagrange equations are:

$$\frac{1}{\alpha} \psi_1'(E_{LK}) \Big[A_{LK} \begin{pmatrix} u \\ v \end{pmatrix} - b_{LK} \Big] = \begin{pmatrix} \mathrm{div}\big(\psi_2'(|\nabla w|^2) \nabla u\big) \\ \mathrm{div}\big(\psi_2'(|\nabla w|^2) \nabla v\big) \end{pmatrix} \tag{3}$$

where $\psi_i'(s^2) = \frac{1}{\sqrt{1 + \frac{s^2}{\beta_i^2}}}$ $i = 1, 2$.

Given that LK-based strategies solve the linear system (1), in order to determine their sources of errors, it suffices to analyze the theoretical assumptions and numerical stability of the system.

2.1 Design Errors (Method Assumptions)

An algorithm is accurate in the measure that it properly models what has to be solved. Otherwise, its solution (even if it has not numerical errors) differs from the problem solution. In the case of LK there are two main aspects that might distort its description of motion.

On the one hand, LK technique is based on the assumption that OF keeps constant in a neighborhood of each pixel. For that, at those locations where there is a collision of different motions, LK can not properly model OF. On the other hand, the matrix A_{LK} corresponds to the structure tensor or second moment matrix [12] and it describes the image local geometry by means of its eigenvalue decomposition. At points with a (unique) well defined orientation, the matrix might be singular (i.e. it is not invertible). This might be the case at straight image contours, specially at horizontal and vertical image edges and flat regions. In contrast, at points with two or more different orientations, the system of equations has a unique solution. The typical case is at corners and junctions. Therefore, LK approach, can not properly solve the aperture problem neither at edges nor flat regions.

2.2 Error Propagation (Numerical Stability)

Errors in the output data that come from errors in the input data are called propagation errors. In the case of OF, the input error is produced by the acquisition of the sequences.

Numerical analysis is a field of mathematics devoted to explore the propagation errors and numerical stability of algorithms. A usual way of handling errors is by setting an upper bound for its relative error. In the case of propagation output errors (ε_{out}) such bound is usually linked to errors in the input data (ε_{in}) by means of a constant K such that:

$$\varepsilon_{out} < \mathrm{K} \cdot \varepsilon_{in} \tag{4}$$

for $\varepsilon_{out} = \|\widetilde{\varepsilon}_{out}\|$ and $\varepsilon_{in} = \|\widetilde{\varepsilon}_{in}\|$ where $\widetilde{\varepsilon}_{out}, \widetilde{\varepsilon}_{in} \in \mathbb{R}^n$ for n being the dimension of the data and $\| \cdot \|$ being a vector norm. The constant K is called condition number and it is an intrinsic property of the algorithm [13]. The condition number might be interpreted as a bound for the fraction of ε_{out} that does not come from ε_{in}. An optimal algorithm (well-conditioned) satisfies $\mathrm{K} \leq 1$. Since this implies that ε_{out} will not be larger than ε_{in} (i.e. $\varepsilon_{out} \in [0, \varepsilon_{in}]$), initial errors are not amplified.

In the case of the solution of a linear system of equations $Ax = b$, the condition number depends on the matrix A and provides an upper-bound in terms of the relative error in b. If we denote ε_{in} as the absolute error in b, then absolute error in x is $\widetilde{\varepsilon}_{out} = A^{-1}\widetilde{\varepsilon}_{in}$ and the quotient between their associated relative errors is

$$\mathrm{K}(A) = \frac{\varepsilon_{out}}{\varepsilon_{in}} = \frac{\|A^{-1}\widetilde{\varepsilon}_{in}\|/\|A^{-1}b\|}{\|\widetilde{\varepsilon}_{in}\|/\|b\|} \leq \|A\|\|A^{-1}\| \quad (5)$$

If we consider the L^2 norm and the matrix A is symmetric (like A_{LK}), the condition number simplifies to [13]:

$$\mathrm{K}(A) = \frac{\lambda_{max}}{\lambda_{min}} \in [1, \infty) \quad (6)$$

where λ_{max} and λ_{min} are the maximum and minimum eigenvalues of A, respectively.

Since high values of confidence measures must be associated to low errors, we propose the following function of K:

$$\kappa(A) = \left(\frac{\lambda_{min}}{\lambda_{max}}\right)^2 \in (0, 1] \quad (7)$$

Notice that now for small values of κ the error propagation might be large, whereas for values near to 1 the error propagation will be small. In case of an indetermination 0/0, $\kappa(A)$ is set to 0. Figure 1 shows the expected correlation between the presented confidence measure and the OF accuracy (EE). Note that it exists a value κ_0 such that, EE is below a threshold EE_0 for all points having $\kappa \geq \kappa_0$. For points with $\kappa \leq \kappa_0$, EE is unbounded, so it uniformly covers the range $[0, \infty)$. It follows that we can use the confidence measure to assess when the error of the computed OF is below a given value.

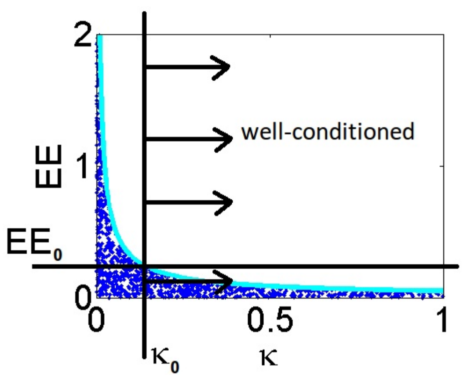

Fig. 1. Theoretical correlation between confidence measure and accuracy

3 Experiments

The goal of the experiments is to check the correlation between confidence measures and OF accuracy. Aside our κ, we have also computed the inverse energy (noted E) reported in [4]. The experiments have been done using the Middlebury database[1] , which contains real-life and synthetic sequences with ground-truth. The parameters for the variational LK implementation[2] are set to $\alpha = 0.02$ and $\beta_i = 0.001$. Flow accuracy is given by the End-point Error (EE) [14].

The dependency between each confidence measure and EE is statistically explored by means of the Spearman correlation coefficient, $\rho \in [-1, 1]$ [15]. It indicates a maximum positive correlation for value 1, and a maximum negative one for value -1. We note that the dependency between any confidence measure and

Table 1. Spearman test for Middlebury sequences

	κ		E	
	ρ	$p-$val	ρ	$p-$val
Dimetrodon	-0.53	$\leq 10^{-3}$	0.55	1
Grove2	-0.62	$\leq 10^{-3}$	-0.48	$\leq 10^{-3}$
Grove3	-0.57	$\leq 10^{-3}$	-0.44	$\leq 10^{-3}$
Hydrangea	-0.69	$\leq 10^{-3}$	-0.43	$\leq 10^{-3}$
RubberWhale	-0.56	$\leq 10^{-3}$	0.16	1
Urban2	-0.63	$\leq 10^{-3}$	-0.40	$\leq 10^{-3}$
Urban3	-0.58	$\leq 10^{-3}$	-0.26	$\leq 10^{-3}$
Venus	-0.60	$\leq 10^{-3}$	-0.06	$\leq 10^{-3}$

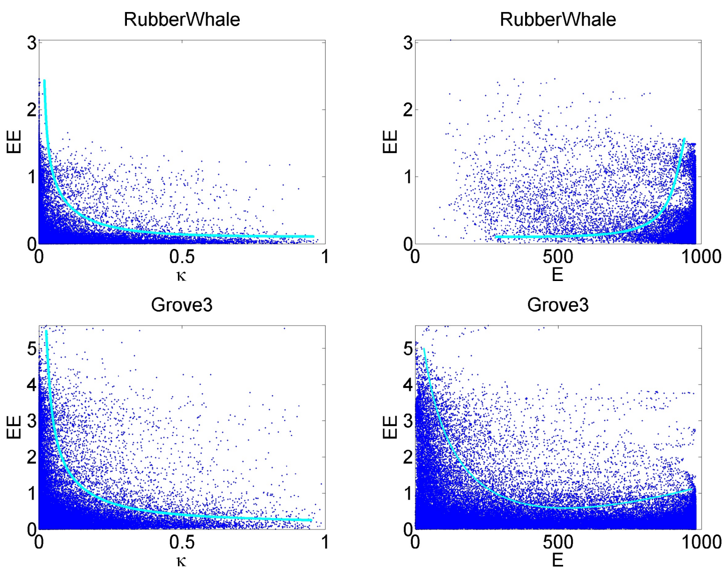

Fig. 2. Point clouds plots for confidence measures κ (left) and E (right) vs EE for the Middlebury sequences RubberWhale and Grove3

[1] `http://vision.middlebury.edu/flow/`

[2] `http://people.csail.mit.edu/celiu/OpticalFlow/`

EE should be decreasing. In order to statistically check it, we have considered the following unilateral hypothesis test:

$$HT : \begin{cases} H_0 : \rho(conf - EE) \geq 0 \\ H_1 : \rho(conf - EE) < 0 \end{cases} \tag{8}$$

for $conf$ the confidence measures κ or E.

Figure 2 plots each confidence measure (x-axis) versus EE (y-axis) and the ideal correlation curve for the Middlebury sequences RubberWhale and Grove3. We observe that, in the case of E, there is not a clear decreasing dependency. Table 1 shows the Spearman coefficient ρ and $p-$values for the test HT. As expected, the Spearman correlation coefficient achieves negative values for κ and the dependency is statistically negative. In the case of the inverse energy E, ρ is not always negative and, even in the cases it is, the dependency is worse than for κ.

3.1 Application to Medical Sequences

In order to illustrate the potential of κ in medical applications, we have applied our confidence analysis to CMR sequences[3]. Figure 3 shows two consecutive

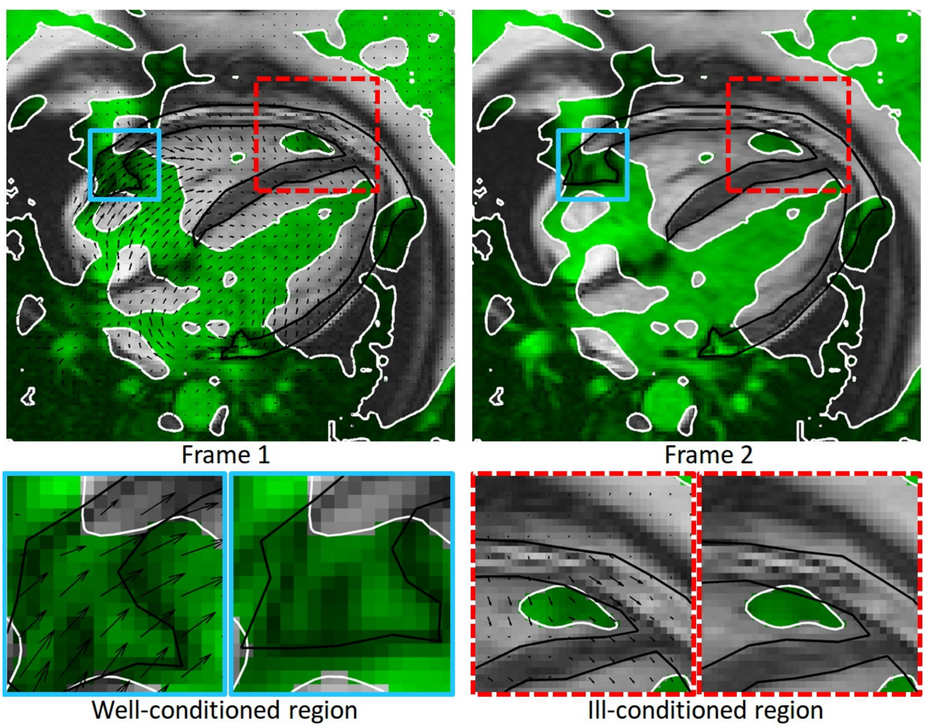

Fig. 3. Application to CMR sequence

[3] Images courtesy of Cardiac Imaging Unit, Hospital de Sant Pau, Barcelona

frames in long axis view with a sudden motion at the basal level. Regions of high κ (above 0.05) are enclosed in green and myocardial walls are outlined in black solid line. Walls in the second frame have been computed by tracking the walls at the first frame using the computed OF depicted in the first frame. Close-ups of well-conditioned and ill-conditioned areas are shown in bottom images. We have chosen a basal area (presenting large motion) for well-conditioned case and a motionless apical one for the ill-conditioned case. The basal wall is correctly tracked, while the apical one does not match the image intensity profile in frame 2 (it transverses the green areas in blood pool).

4 Conclusions and Future Work

In spite of the advances in the design of variational schemes, confidence measures are rarely addressed in the literature. However, they are essential to decide in which regions the computed flow field is reliable. This paper reports an analysis of the numerical stability of LK based schemes and presents a confidence measure correlated with OF accuracy. Experiments on CMR sequences show that a drop in our confidence measure implies an erratic random direction of the computed OF, while high values ensure stable and coherent flows.

There are some improvements that should be done in the near future. In order to ensure a practical application, we should statistically determine which is the minimum value of the confidence measure ensuring a given predefined accuracy. We also plan to exhaustively compare κ to existing confidence measures such as local structure based, energy based or statistically based.

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