ANISOTROPIC CONTOUR COMPLETION

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ABSTRACT

In this paper we introduce a novel application of the diffusion tensor for anisotropic image processing. The Anisotropic Contour Completion (ACC) we suggest consists in extending the characteristic function of the open curve by means of a degenerated diffusion tensor that prevents any diffusion in the normal direction. We show that ACC is equivalent to a dilation with a continuous elliptic structural element that takes into account the local orientation of the contours to be closed. Experiments on contours extracted from real images show that ACC produces shapes able to adapt to any curve in an active contour framework.

1. INTRODUCTION

The main goal of any image segmentation/interpolation process consists in approaching a set of unconnected points that conform to certain characteristics. A usual way of modelling uncompleted shapes is by means of a snake [1], [2], [4]. Snakes are curves that minimize an energy functional. Both in the case of parametric snakes [4] and geodesic active contours [1], [2], this energy consists of an internal energy that confers smoothness to the model and an external energy depending on the contour to be approached. Since the snake deforms by means of the gradient descent of the energy functional, the definition of the external energy is crucial for a successful segmentation.

The most commonly used external energies are the distance map to the contour of interest and a decreasing function of the norm of the image gradient. In the first case, creasts of the distance map, produced by the geometric features of the zero level curve that represents the contour, induce local minima that may trap the snake at wrong models. In the second, the external force scope reduces to a narrow neighborhood of the image edges. This forces either an initial snake close to the edges of interest or an extension with Gradient Vector Flow (GVF)/Generalized Gradient Vector Flow (GGVF) [7], [8]. The first solution implies manual intervention, highly undesirable in automated procedures. The second, the most effective up to our knowledge, produces a smooth extension of the gradient of the image edge map that attracts snakes to a large variety of shapes. However both GVF and GGVF are based on an isotropic linear process (heat diffusion) which makes the vector field have saddle points, when edges are highly non-convex, that prevent snakes from entering into concave regions.

A completely different approach to contour closing comes from mathematical morphology [5] [9]. A dilation at a suitable scale produces a complete curve that can serve as initial snake in a deformable model process. The associated structural elements (typically, lineal or circular) are constant. This constitutes a main inconvenience, since it may imply that the closed shapes differ significantly from the incomplete contours. The proper way of closing contours should use elliptic structural elements with principal axis oriented in the tangent direction of the curve to complete.

Dilations are the geometric way of extending functions (the characteristic function of the open curve in our case). In this paper, we will see that functional extension is governed by a parabolic Partial Differential Equation (PDE). The differential equation equals that of a diffusion process except for the boundary conditions. The most studied parabolic PDE's are those that describe heat diffusion. The process has naturally associated a metric, given by the diffusion tensor, that locally describes the way heat extends or distributes. Thus an anisotropic heat diffusion is the analytic way of handling a dilation with non-constant elliptic structural elements. The closing procedure we propose is based on an anisotropic extension of functions. By extending the characteristic function of the opened contour in the direction of maximum coherence [10], we succeed in recovering a reliable closed model of the curve. Its proximity to the contour of interest assures its convergence when used as initial snake in a deformable contour process.

2. INFORMATION EXTENSION

Diffusion is the natural physical way of distributing information. The dynamic process of the evolution of an initial

This work is partially supported by the "Ministerio de Ciencia y Tecnologia" grant TIC2000-1635-C04-04.

heat distribution, u_0, in time is governed by a second order parabolic PDE that can be written in divergence form:

$$u_t(x, y, t) = \mathrm{div}(J\nabla u) \tag{1}$$

The solution u represents the heat or mass distribution in the plane at each time t. The diffusion tensor, J, is a symmetric and positive defined matrix, that is, a metric. Geometrically, a metric is described by means of an ellipse with principal axes of lengths equal to the eigenvalues of J (λ_1, λ_2) oriented by its corresponding eigenvectors ξ and η. The ellipse associated to this metric locally describes the way heat distributes in the plane: an amount λ_1 of heat travels along ξ and an amount λ_2, in the direction η. Notice that setting to zero one of the two eigenvalues, we restrict diffusion to the integral curves of the vector field with the positive eigenvalue. We will make use of this fact in Section 2.2.

In the context of metrics, diffusion processes are classified into **isotropic**, when the eigenvalues of J are equal (circular structural elements) and **anisotropic**, in the case of different eigenvalues (elliptic structural element).

Concerning the final heat distribution, steady states of (1) can be described by means of their level sets. If we denote by Ω the region enclosed by a level curve γ then, Stoke's Theorem, yields that final states must satisfy the following integral equation:

$$0 = \int_\Omega \mathrm{div}(J\nabla u) = \int_\gamma \frac{\nabla u^t J \nabla u}{|\nabla u|} \tag{2}$$

Therefore the final heat distribution will be constant unless the metric given by J degenerates (i.e. cancels) on some closed curves. In this case, the final heat distribution will consist of closed regions of uniformly distributed heat separated by these curves. In thermodynamic terms we may think that these curves behave like insulators.

This property is commonly used in image processing. The original image is set to be the initial heat distribution and the metric is chosen in such a way that it degenerates on points that satisfy some conditions. In this manner the final state that we achieve is an image so that features of interest in the original image are easier to identify.

2.1. Extension of a function on the image domain

Heat diffusion has another mathematical and physical use hardly exploited in image analysis. Heat diffusion (second order elliptic operators, in general) has the property of smoothly extending a function defined on a curve in the plane. If L denotes an elliptic operator, then the function that solves the PDE:

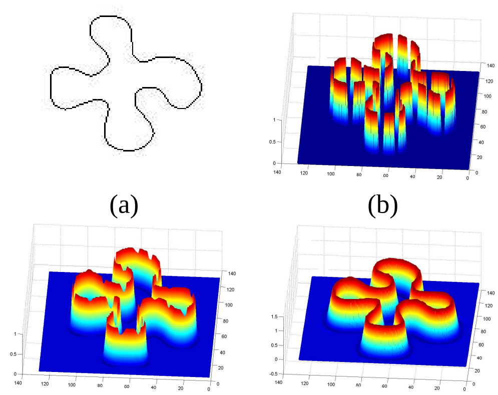
$$\begin{aligned} Lu &= 0 \\ u_{|\gamma} &= f \end{aligned}$$

is the unique smooth extension [3] of the function f which was defined only on the curve γ. The equation is solved by seeking the steady states of the associated parabolic PDE:

$$\begin{aligned} u_t &= Lu \\ u_{|\gamma} &= f \end{aligned} \tag{3}$$

From the point of view of thermodynamics, we may think that the heat distribution on γ given by f never puts out. We will restrict to elliptic operators admitting a divergence form given in equation (1). The ellipse describing the metric given by J corresponds to the structural element of the associated dilation. In the case of classic mathematical morphology, the extension is based on the Laplacian operator and the function to be extended is the characteristic function of a set of points. The scale or radius of the dilation corresponds to time in equation (3).

By using anisotropic heat operators we can control the direction towards information is extended. This can be used to complete unconnected contours as follows.



(a) (b) (c) (d)

Fig. 1. Function extension: clover (a), image graph of uncomplete clover (b), intermediate step (c) and closing (d).

2.2. Anisotropic Completion of Contours

There are two privileged vector fields in an image u, the unit normal to the level curves, $\frac{\nabla u}{|\nabla u|}$, and its unit tangent, $\frac{\nabla u^\perp}{|\nabla u|}$. The eigenvectors of the Structure Tensor [10], η and ξ, are a robust way of computing the unit normal and tangent of the level curves. If we consider a metric $\tilde{J}$ with eigenvectors η and ξ and eigenvalues $\lambda_1 = 0$ and $\lambda_2 = 1$, then equation (2) tells us that closed contours of the initial image are preserved during the evolution. Meanwhile for incomplete α-level curves, the effect of distributing heat only in the tangent direction, makes these curves evolve towards a closed contour of uniform gray level.

Therefore if we use this anisotropic process to extend a binary map of a unconnected curve (i.e. its characteristic function), the final state will be a binary map of a closed model of the uncomplete initial contour. Intuitively, the final closed shape that we recover resembles the one we would get if we drew the tangent at the boundary points of the original curve and intersected the lines. This process is the Anisotropic Completion of Contours we suggest :

$$\begin{aligned} u_t &= \operatorname{div}(\tilde{J}\nabla u) \\ u_{|\gamma} &= u_0 \end{aligned} \tag{4}$$

with u_0 the characteristic function of the opened contours, γ, and the diffusion tensor $\tilde{J}$ as described in the previous paragraph. Figure 1 illustrates the different stages in the process of gap filling for an incomplete clover.

Notice that the iterative Euler numeric scheme used to integrate equation (4), admits a stop criterion in terms of the magnitude of $\operatorname{div}(\tilde{J}\nabla u)$, as we know that the evolution converges to an image with a closed contour.

3. APPLICATIONS TO IMAGE SEGMENTATION

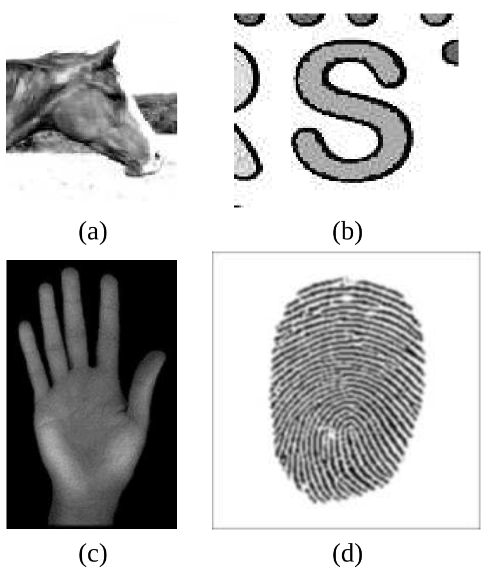


Fig. 2. Test images: horse (a), character 'S' (b), hand (c) and fingerprint (d).

We base image object segmentation/interpolation in the approximation of a set of (possibly unconnected) points that lie on the object we want to model. We consider that the object has been successfully segmented once we have a closed contour approaching this set of unconnected points. Although snakes are an alternative approach, poor convergence of snakes to concave shapes obliges an initial snake close to the curve to be modelled. We propose the following strategy to model uncompleted contours. First we apply ACC to the characteristic function of the selected set of points in order to produce a closed contour. The crests of the extension are an approximation to the shapes of interests reliable enough as to ensure convergence in a deformable contour process, when set to be the initial snake.

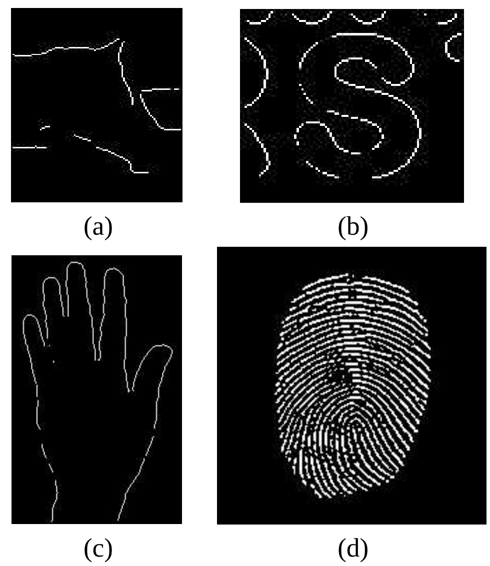


Fig. 3. Open contours: horse (a), character 'S' (b), hand (c) and fingerprint (d).

Figure 2 displays the set of images used. Notice that the set of points on the object of interest corresponds to those image pixels that conform to given characteristics. We chose edges for the character 'S', the hand and the horse and valleys in the case of the fingerprint. The characteristic functions that ACC will extend are shown in figure 3. In all cases the structure tensor was computed using a gaussian of variance $\rho = 2$ over an image gradient regularized with a gaussian of variance $\sigma = 1.5$.

The closings obtained with ACC are displayed in figure 4. If there is more than one object, ACC is stopped at a fix number of iterations, which depends on the degree of incompleteness of the contours. In the case of the horse's head extension was stopped after 800 iterative steps. The other extensions were obtained either by means of the magnitude of $\operatorname{div}(\tilde{J}\nabla u)$ or when the set of gray level one has only two components. The result of applying a dilation with a circular element is depicted in figure 5. Classic dilations failed to obtain a closed curve and produced shapes which may not resemble the original uncomplete curves. In this context, the character 'S' and the horse's head are extreme pathological cases. In the first case (fig.5 (a)), the right hand side square has been included as part of the horse's head. In the second, concavities (fig.5 (b)) have been swallowed by the final, still uncompleted, shape. Comparing both models,

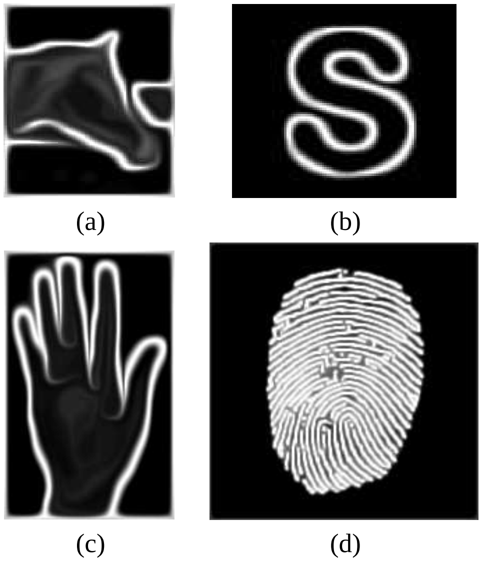


Fig. 4. ACC closings: horse (a), character 'S' (b), hand (c) and fingerprint (d).

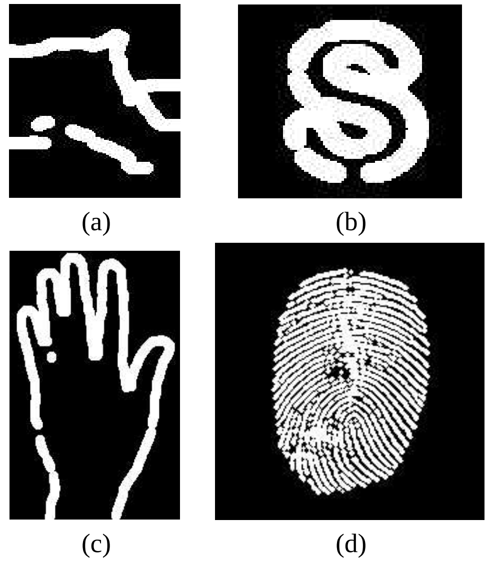


Fig. 5. Classic dilation: horse (a), character 'S' (b), hand (c) and fingerprint (d).

contours computed with ACC (fig.4) are more accurate and conform to the underlying shapes of the open curves.

4. CONCLUSIONS AND FUTURE WORK

In the present paper we have introduced extension of functions governed by a diffusion tensor as a tool to close uncompleted contours. We have shown that the ellipse describing the metric associated to the diffusion tensor corresponds to the shape of the morphological structural element. In this context, we have developed a novel curve closing approach. Completion of contours is achieved by an anisotropic extension of the characteristic function of the contour to be closed. In this manner the structural element of the corresponding dilation takes into account the local orientation of the uncompleted contours. This produces closed model of curves more reliable than the ones obtained with classical isotropic dilation. This makes ACC useful in image segmentation as a tool to obtain good initializations for a snake.

Our current efforts focus on extending ACC to noisy images. To such purpose, image coherence (computed in terms of the diffusion tensor eigenvalues) will be taken into account in order to choose the structures to be extended.

5. REFERENCES

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