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A Riemannian Approach to Cardiac Fiber Architecture Modelling

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ABSTRACT

There is general consensus that myocardial fiber architecture should be modelled in order to fully understand the electromechanical properties of the Left Ventricle (LV). Diffusion Tensor magnetic resonance Imaging (DTI) is the reference image modality for rapid measurement of fiber orientations by means of the tensor principal eigenvectors.

In this work, we present a mathematical framework for across subject comparison of the local geometry of the LV anatomy including the fiber architecture from the statistical analysis of DTI studies. We use concepts of differential geometry for defining a parametric domain suitable for statistical analysis of a low number of samples. We use Riemannian metrics to define a consistent computation of DTI principal eigenvector modes of variation. Our framework has been applied to build an atlas of the LV fiber architecture from 7 DTI normal canine hearts.

Key Words: *cardiac fiber architecture, diffusion tensor magnetic resonance imaging, differential (Riemannian) geometry.*

1 INTRODUCTION

Cardiologists accept that analysis of myocardium motion, especially the Left Ventricle (LV), provides information about the health of the heart. Since the architecture of myocardial fibers determines LV electromechanical activation pattern, as well as, its mechanics, its thorough knowledge is crucial for defining reliable computational models. Diffusion Tensor Magnetic Resonance Imaging (DTI) has the ability of measuring the diffusion of water molecules along various directions in tissues. This converts DTI volumes in a unique (medical) imaging modality for visualizing the local structure of fibrous tissue. In the particular case of myocardial fibers, it is well established that the primary eigenvector is locally aligned with fiber direction [1].

In this work, we introduce a unifying mathematical framework for computing a statistical atlas of fiber orientations from the analysis of DTI studies. We define differentiable charts [2] parameterizing the LV volume in unitary (radial, longitudinal and circumferential) coordinates (the Normalized Parametric Domain, NPD). In order to ensure registration across subjects, the parametric coordinates are defined according to common anatomic landmarks.

By differential geometry arguments [2] the parametric map defines (local) coordinates on the LV spatial volume and, by means of the Jacobian, on its tangent space. The local reference system given by the

Jacobian describes the geometry of the LV anatomy and it is used to decompose DTI primary eigenvector. The components of the DTI principal eigenvectors in the local reference system are mapped onto the NPD for a PCA analysis of their variability. Riemannian metrics are used to provide a consistent computational framework for variation modes.

2 MAIN BODY

In order to quantify the variability of ventricular geometry and fiber architecture of the LV across subjects, the data volumes of the different subjects should be registered first. Current approaches [3-4] use a non-rigid deformation based on, both, image intensity and anatomical landmarks to register data volumes to the image volume of a reference subject. A common inconvenience is that registration does not provide any geometric description of the LV anatomic shape. Such description is incorporated by a global coordinate change representing (parametrizing) the geometry of an approximate template. Usual coordinate changes (such as cylindrical or prolate spheroidal) model an oversimplified geometry unable to account for the patient-specific (local) anatomic form of the heart.

As suggested in [5], we parameterize the LV volume ($\mathcal{LV}$) in normalized circumferential, longitudinal and radial directions (Fig. 2 (a)) using 3D B-Splines. The parametric coordinates define for each subject a mapping, $\Psi : \Omega^3 = [0,1] \times [0,1] \times [0,1] \longrightarrow \mathcal{LV}$, between the unitary cube and any $\mathcal{LV}$ domain. We call Ω^3 Normalized Parametric Domain (NPD). By taking into account anatomic features common to any LV, we ensure that given anatomic locations in $\mathcal{LV}$ share the same parametric configuration $(u, v, w) \in \Omega^3$. In this manner, we achieve implicit registration among different subjects by means of the inverse of the parametric mapping Ψ^{-1}.

The map Ψ registers $\mathcal{LV}$ to the unitary (cubic) domain Ω^3 "straightening" (unwrapping) $\mathcal{LV}$ geometry. A main advantage over approaches registering volumes to a reference $\mathcal{LV}$ domain in cartesian coordinates is that the NPD domain allows a straightforward definition of neighborhoods adapted to $\mathcal{LV}$ subject-specific anatomy.

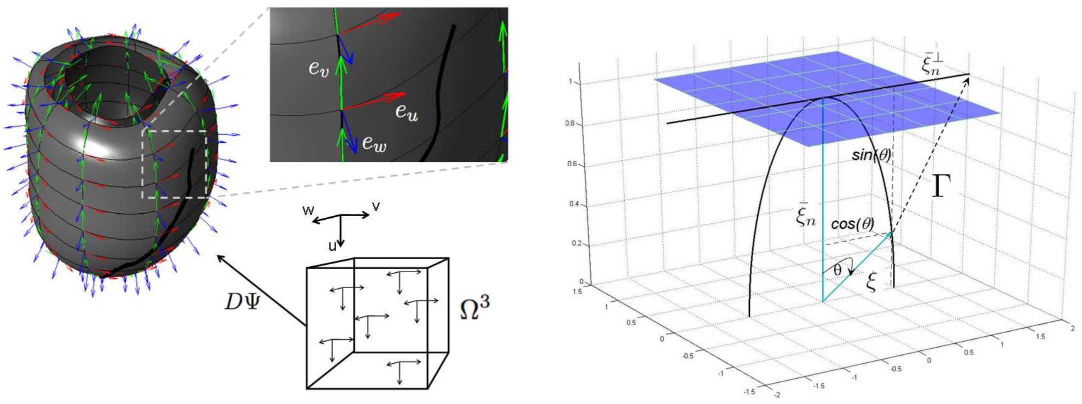
The Jacobian, $D\Psi(u, v, w)$, of the parametric map defines at each point $p \in \mathcal{LV}$ a non orthogonal reference of unitary vectors $\{e_u(p), e_w(p), e_w(p)\}$:

$$e_u = \frac{\nabla_u \Psi}{\|\nabla_u \Psi\|_2}, e_v = \frac{\nabla_v \Psi}{\|\nabla_v \Psi\|_2}, e_w = \frac{\nabla_w \Psi}{\|\nabla_w \Psi\|_2} \tag{1}$$

describing the local geometry of the $\mathcal{LV}$ volume. Unlike approaches using a global change of coordinates in the cartesian image volume, our local reference is able to capture the subject-specific $\mathcal{LV}$ geometry (given by the parametrization of the volumetric manifold [2]). Figure 2 shows the description of $\mathcal{LV}$ local geometry given by $D\Psi(u, v, w)$. We observe that the reference vectors (1) are tangent to $\mathcal{LV}$ parametric curves, so that in the NPD they correspond to the axis defined by the parametric coordinates (u, v, w).

The tangent application $D\Psi(u, v, w)$ maps vectors, ξ, expressed in cartesian image volume coordinates into the NPD [2]. By linearity of the tangent application, the mapping is given by the decomposition of ξ in the local reference (1). Such components can be mapped to the NPD for statistical analysis.

The atlas of myocardial anatomy includes a mean geometry of the $\mathcal{LV}$, as well as, exploring the principal modes of variation of the principal eigenvector average orientation. The average $\mathcal{LV}$ template follows from the 3D B-Spline parametrization of the average position of points of different subjects obtained by uniform sampling of the parametric space. Concerning fiber architecture, the computation of an statistical atlas requires statistical measurements (arithmetic mean and covariance matrix) on the values (ξ_u, ξ_v, ξ_w) for all subjects. In order to compensate for the low number of DTI studies, N, for each parametric point (u_i, v_j, w_k), we considered the values (ξ_u, ξ_v, ξ_w) in a 6-connected neighborhood



(a) (b)

Figure 1: Local Reference System given by the Parametric Map Describing $\mathcal{LV}$ Geometry, (a), and sketch of the the definition of the isometry on S^2 for statistical analysis of DTI principal eigenvector.

defined in the NPD. This strategy increases to $7N$ the number of samples for computing the descriptive statistics. We will note the set of samples for each parametric point Ξ.

For an arbitrary vector, (ξ_u, ξ_v, ξ_w) would be a point cloud in $\mathbb{R}^3$. In our particular case, DTI principal eigenvector in cartesian coordinates is unitary (i.e. it belongs to the sphere, S^2). Although the local reference system (1) is non-orthogonal (i.e. the tangent application does not preserve the metric), it does not significantly deviate from orthonormality. This implies that $(\xi_u, \xi_v, \xi_w) \in \Xi$ approximately lie on S^2 and, thus, the statistical analysis should not be done in $\mathbb{R}^3$ but on the Riemmanian manifold S^2 [6].

In the particular case of the sphere, it is possible to construct an isometry mapping S^2 onto a plane such that distances can be measured using the Euclidean metric in the plane. Let $\bar{\xi}$ be the average of all $\xi = (\xi_u, \xi_v, \xi_w) \in \Xi$ and $\bar{\xi}_n = \bar{\xi}/\|\bar{\xi}\|$ the associated unitary vector. The inverse of the exponential map [6] projects maximum circles through $\bar{\xi}$ to its perpendicular vector, $\bar{\xi}_n^{\perp}$, on the tangent plane (see fig.2 (b)). By general theory of Lie groups the exponential map is a local isometry. In the particular case of spheres [6], it is an isometry between the circle and the vector space generated by $\bar{\xi}_n^{\perp}$ given by the angle, θ, between $\bar{\xi}_n$ and any point ξ in the maximum circle:

$$\exp^{-1} : \xi \longmapsto \theta \bar{\xi}_n^{\perp} \tag{2}$$

By trigonometric arguments $\bar{\xi}_n^{\perp}$ and θ are given by (see fig.2 (b)):

$$\bar{\xi}_n^{\perp} = \frac{\xi - \langle \xi, \bar{\xi}_n \rangle}{\|\xi - \langle \xi, \bar{\xi}_n \rangle\|} \quad \theta = \arctan\left(\frac{\langle \xi, \bar{\xi}_n^{\perp} \rangle}{\langle \xi, \bar{\xi}_n \rangle}\right) \tag{3}$$

By definition the map (2) provides three values which correspond to coordinates in the NPD. It follows that their statistical analysis can be still anatomically interpret in terms of the local reference (1). A Principal Component Analysis for $\exp^{-1}(\xi)$, $\xi \in \Xi$, gives the fiber average model and its modes of variation. Regarding the modes of variation, since $\exp^{-1}(\xi)$ are on a plane, the covariance matrix always has a zero eigenvalue corresponding to the direction perpendicular to the plane $\bar{\xi}_n$. The remaining modes are in terms of the local reference (1) and can be anatomically interpreted.

Our mathematical framework has been applied to DTI studies of $N = 7$ normal canine hearts from the Johns Hopkins Hospital public data based (available at http://www.ccbm.jhu.edu/). The NPD has been sampled in $100 \times 50 \times 10$ parametric points uniformly distributed. Figure 2 shows the average fiber model over the average geometry of the $\mathcal{LV}$. Vectors are colored according to the sign of the circumferential component: cyan stands for positive orientation and black for negative one.

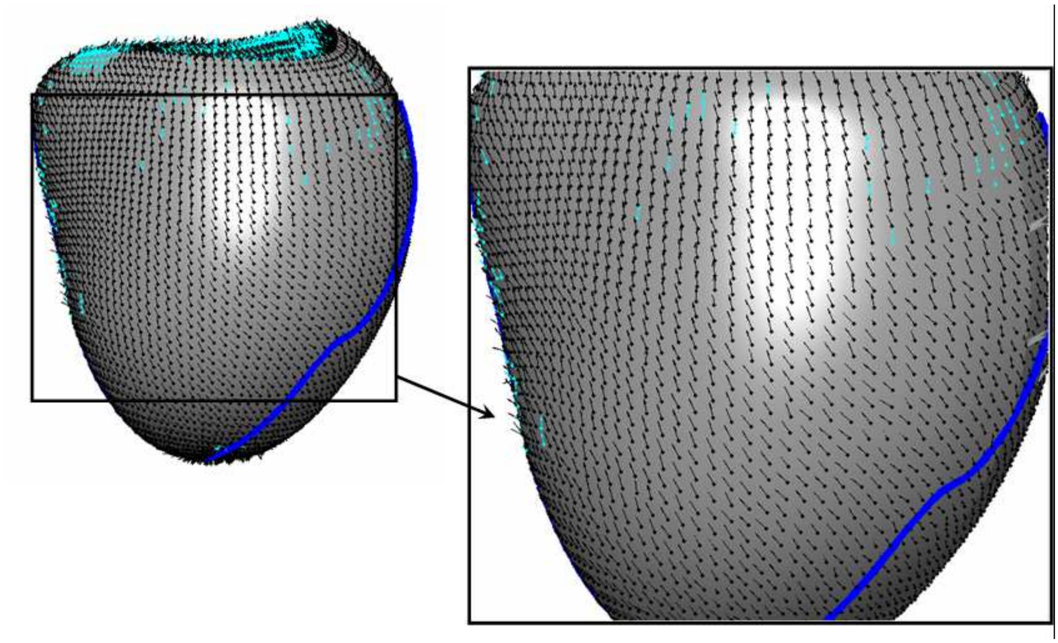


Figure 2: Average fiber architecture over an average geometry of the $\mathcal{LV}$ anatomy.

3 CONCLUSIONS

Modelling the architecture of cardiac fibers is a challenging problem and a milestone for developing consistent models of the heart electromechanical activation. On the grounds of Riemannian theory, we have presented a general mathematical framework for computing consistent statistical models of cardiac fiber architecture from the analysis of DTI images. The average model is suitable for fiber tracking and simulation of heart electromechanical propagation models [7], whereas the modes of variation can be anatomically interpreted.

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