Endowing Canonical Geometries to Cardiac Structures

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Abstract. In this paper, we show that canonical (shape-based) geometries can be endowed to cardiac structures using tubular coordinates defined over their medial axis. We give an analytic formulation of these geometries by means of B-Splines. Since B-Splines present vector space structure PCA can be applied to their control points and statistical models relating boundaries and the interior of the anatomical structures can be derived. We demonstrate the applicability in two cardiac structures, the 3D Left Ventricular volume, and the 2D Left-Right ventricle set in 2D Short Axis view.

1 Introduction

The Myocardium presents two main attributes when seen in medical images: shape and appearance. The shape of the volume enclosed by myocardial walls is an intrinsic attribute that represents its anatomy and is easily captured by different anatomical imaging modalities such as CT or MR. In contrast, the appearance is image-dependent given that different modalities and/or acquisition protocols yield different (complementary) physiological information(perfusion, motion or fiber architecture). Statistical analysis of both attributes is a powerful tool that provides good indicators for disease diagnosis, progression and therapy planning and should handle complementary information coming from different sources.

First attempts to relate shape and appearance features were pioneered by Cootes *et al.* in order to improve their Active Shape Model (ASM) search [1]. They considered appearance patterns by sampling image intensities along fixed-length profiles projected orthogonally from the boundary. However, this approach was not able to capture the whole appearance contained inside the object boundaries. In order to solve this, the same authors later introduced Active Appearance Models (AAM) [2], that relate the whole object appearance and its boundary. However, since AAMs are based on warps registering images, they do not provide a canonical way of performing this relation.

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A natural way of associating shape and appearance attributes on the whole anatomic domain is by means of a parametrization (in the manifold sense) providing coordinates, inside the object, related to the boundary anatomy. Endowing geometry to shape models is achieved by using representations explicitly describing the structure geometry, like medial axis representations [3, 4, 5] or shape parametrizations [6]. Medial axis representations (*m-reps* [5]) describe geometry using the centres and radius of osculating spheres. Since osculating spheres are those interior spheres having more than one point tangent to the object boundary, they describe geometry by means of the structure boundary principal curvatures. However, *m-reps* present two main inconveniences. On one hand, they are discrete representations which might decrease accuracy of the model [7]. On the other hand, they do not provide a direct description of geometry and have a non-vectorial structure, which requires statistics in Riemmanian spaces [4].

The framework of differential geometry provides suitable tools for describing shape geometry by means of object parametrization in the manifold sense. A main shortcoming is that parametric maps are an infinite dimensional space without vector structure. Although there are a number of authors who have contributed to statistical analysis on Riemmanian manifolds [8], in the infinite case, computation of descriptive statistics is a delicate step not thoroughly solved [9]. A practical way of approaching parametrization of anatomical structures for shape analysis is by using basis functions for the formulation of the parametric map. In this manner, the space of parametric maps is vectorized and PDM approaches serve to compute statistical models. One of the first works is the spherical harmonic (SPHARM) parametrization for the hippocampus modelling used in [6]. The framework is suitable for structures diffeomorphic to spheres (i.e. admitting angular parameters), but the methodology does not generalize to more complex cardiac structures (like the right-left ventricle set, homeomorf to a double torus).

A more recent approach are *cm-reps* models [10] which are continuous explicit generalizations of *m-reps*. The parametrization of the shape is based on a parametrization of the medial axis which is extended to the whole domain by an inverse skeletonization process. In this manner, shape-based ("natural") coordinates are consistently defined over the anatomical structures. These coordinates allow to establish correspondences between structures across different subjects, and also allows to map intensities in these structures into a canonical (fixed for all subjects) reference frame in which shape differences between subjects have been effectively removed. Since these coordinates are defined by finding an analytical relationship between the structure medial axis (skeleton) and its boundaries, the framework naturally handles the combined analysis of shape and appearance. In its current formulation, the methods has two shortcomings. The inverse skeletonization requires solving a biharmonic PDE with nonlinear boundary conditions, which implies a high computational cost. In addition, this approach only provides well defined coordinates along the radial direction, but not in the medial surface manifold, which is represented as a mesh. We consider, that endowing shape based coordinates to a biological structure should apply

to the whole anatomy, especially in the cardiac context where 3 main directions naturally arise: circumferential, radial and, in the 3D case, longitudinal.

In this work, we transfer the philosophy of the combined analysis of shape and appearance hold in *cm-reps* to cardiac imaging. That is, we define shape-based coordinates to 2D and 3D cardiac structures by endowing them with a canonical geometry. This geometry is obtained using tubular coordinates given by normal sections to the medial axis of the structure that, in turn, is parametrized in a consistent manner, taking into account anatomical features. We show the potential of the proposed framework by modelling the shape of the left ventricle including the basal ring and the right and left ventricle joined geometry in 2D short axis views. In this last case, the left-right ventricles set, that is not a "simple object".

2 Differential Geometry Background

An $n-$dimensional differential manifold, $\mathcal{M}$, can be thought as the result of doing patchwork. That is, it can be made by "cutting" pieces of $\mathbb{R}^n$, "deforming" them and smoothly "gluing" them together until the geometric shape is covered. Mathematically, this states that there exists an open covering of the manifold, $(U_\alpha)_{\alpha \in A}$ homeomorphic to $\mathbb{R}^n$ via bijective continuous maps:

$$\begin{array}{rcl} \Phi_\alpha : \mathcal{U}_\alpha & \longrightarrow & U_\alpha \subset \mathbb{R}^n \\ \mathbf{x} = (x_1, \ldots, x_n) & \longmapsto & \mathbf{u}(\mathbf{x}) = (u_1(\mathbf{x}), \ldots, u_n(\mathbf{x})) \end{array}$$

such that, for any two indexes α, β, the composition $\Phi_\alpha \circ \Phi_\beta^{-1}$ is differentiable. The pair $(\mathcal{U}_\alpha, \Phi_\alpha)$ is called *local chart* or *local coordinate system*, $U_\alpha = \Phi(\mathcal{U}_\alpha)$ is called parametric domain and $\Psi_\alpha = \Phi_\alpha^{-1}$ parametrization.

The set of local charts endows the manifold with a topology (i.e., neighbours). The geometry arises with the definition of directions (e.g., left-right, up-down) in each open neighbourhood. Directions in differentiable manifolds are given at each point $\mathbf{x}$ by its tangent space. The elements of the tangent space are called tangent vectors at $\mathbf{x}$ and, intuitively, they describe all possible "directions" through $\mathbf{x}$.

The tangent space is given by the columns of the Jacobian matrix and has dimension n. The vectors perpendicular to tangent vectors are a vector bundle of dimension $d = m - n$ called normal space.

The normal space of a differentiable manifold defines tubular coordinates in $\mathbb{R}^m$ around $\mathcal{M}$ by means of normal sections. If we denote $\overrightarrow{n}_{\mathbf{x}}$ the normal space at $\mathbf{x}$, tubular coordinates are defined as:

$$\begin{array}{rcl} U_\alpha \times V_\beta \in \mathbb{R}^n \times \mathbb{R}^d & \longrightarrow & \mathbb{R}^m \\ (\mathbf{u}, \mathbf{r}) & \longmapsto & \Psi(\mathbf{u}) + \mathbf{r}\overrightarrow{n}_{\Psi(\mathbf{u})} \end{array}$$

for $\mathbf{r}\overrightarrow{n}_{\Psi(\mathbf{u})} := \sum_{i=1}^{d} r_i \overrightarrow{n}_{\Psi(\mathbf{u})}$. That is, for each point $\Psi(\mathbf{u})$ we can move on its normal direction along radial coordinate. It follows that, by means of radial coordinates, we have a distance map to $\Psi(\mathbf{u})$.

3 Canonical Coordinates over Anatomical Manifolds

Anatomical structures define volumetric domains in the ambient space which admit tubular coordinates by means of their medial axis. The medial axis is given by interior points equidistant to two or more boundary points [3]. It follows that its associated tubular coordinates parameterize the anatomical volume.

In the case of anatomical structures, their medial axis is a compact manifold that might be parametrized in angular coordinates. It follows that, by periodicity, the tubular coordinates change is given in a single chart covering the whole structure volume.

In order to provide a canonical geometry, tubular coordinates might be defined such that two manifolds, $\mathcal{M}_1, \mathcal{M}_2$, representing the same anatomical structure, share a common (fixed) parametric domain.

Anatomical structures present several landmarks common to any subject and easily identifiable. The implicit registration is achieved by assigning to anatomical landmarks normalized tubular coordinates codifying their position relatively to the geometry of the organ. We note that, in this manner, parametric coordinates have an anatomical meaning.

The parametrization map Ψ can be analitically approximated by means of basis functions. In our case we choose m-dimensional B-Splines since they are easy to implement and computationally efficient:

$$\Psi(\mathbf{u}, \mathbf{r}) = \sum_{J}^{M_J} \sum_{I}^{M_I} B_I(\mathbf{u}) \cdot B_J(\mathbf{r}) \cdot P_{IJ} \tag{1}$$

Here, $\mathbf{u}$ and $\mathbf{r}$ are the medial axis and the tubular parameters respectively.

Although B-Splines are not as general as, for instance, NURBS, they provide enough flexibility for the biological structures considered in this work.

Parametrizations expressed in terms of basis functions present vector space structure. Thus, the components in the chosen basis can be statistically analysed by means of standard PCA. In our case, PCA is applied to the control points of the B-Spline.

The tubular parametric map given by (1) is computed in two steps:

1. **Medial axis parametrization.** It corresponds to setting $r = 0$ in equation (1). Therefore, it is defined as soon as anatomical coordinates ensuring implicit registration are assigned to points on the medial axis. Such requirement is fulfilled by assigning given angular ranges to curvature extremum and junctions on the medial axis. Once we have pairs $(\mathbf{x}, \mathbf{u})$ of points and their corresponding parameters, the B-spline parametrization is obtained by minimizing:

$$\sum_{i=1}^{N} \|\Psi(\mathbf{u}_i, 0) - \mathbf{x}_i\|^2 \tag{2}$$

2. **Volume parametrization.** The parametric map for $r > 0$ is defined by extending the coordinate $\mathbf{u}$ along radial directions. Since radial coordinates correspond to the distance to the medial axis, $\Psi(\mathbf{u}_i, 0)$ is extended by means

of the distance map to the medial axis. By definition of medial axis, this is equivalent to deforming a B-spline snake to fit the structure boundaries. In order to ensure the implicit registration requirement, unitary radial parameters ($\|\mathbf{r}\| = 1$) are assigned to points on the structure boundaries.

4 Application to Cardiac Structures

We apply our framework to provide a canonical geometry in two cardiac structures of interest, the Left-Right Ventricle set (LV/RV) seen in 2D Short Axis (SA) slices, and the 3D LV volume. In both cases we use LV-RV junctions as landmarks and, in the 3D case, we also consider the apical cap and the basal ring. These landmarks are used to define an affine anatomic reference $\{O; V_x, V_y, V_z\}$ in order to remove variability in subject-device relative position as follows:

V_z is defined as the tangent vector to the line passing through the apical cap, A, and the centroid of the endocardial basal ring, B: $V_z = (B - A)/\|B - A\|$. The positions of A and B manually are determined in Long Axis (LA) views. The origin is defined along V_z-axis as: $O = A + 2/3(B - A)$ in order to account for any translation among different subjects. The vector V_x, is a unitary vector starting at O and pointing to the junction of the right and left ventricles the septum and the inferior walls. Since V_x points the same anatomical location for any LV, by setting V_x as the origin of angles, we handle any rotational disparity among different subjects. The vector V_y is chosen to make $\{V_x, V_y, V_z\}$ a negatively oriented orthonormal system. Figure 1 illustrates the orientation of the anatomical affine reference and the anatomical landmarks. Stars indicate the right and left ventricle junctions used to orient the reference in short axis views.

4.1 3D Left Ventricle Including the Basal Ring

The medial axis of the left ventricle volume is a surface that it is diffeomorphic to the sphere. Thus, it can be parametrized using circumferential and longitudinal

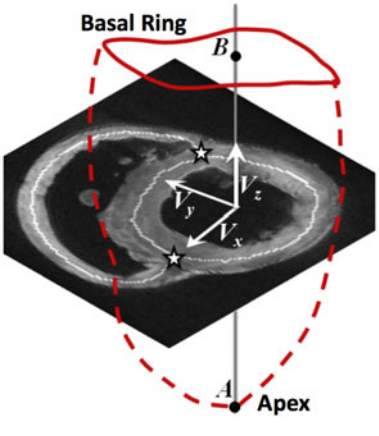


Fig. 1. Anatomical Affine Reference and Landmarks of the Myocardium

coordinates. We note (u, v, r) the parametric coordinates standing for the circumferential, longitudinal and radial directions.

Parameter directions on the medial axis are assigned as follows. The circumferential parameter u is assigned by mapping the same circumferential range $[0, u_S]$ to non-septal segments and the complementary $[u_S, 2\pi]$ to the septal one:

$$u = \begin{cases} \frac{u_S}{\theta_S}\theta, & \theta \leq \theta_S \\ \frac{2\pi - u_S}{2\pi - \theta_S}\theta + 2\pi\frac{\theta_S - u_S}{2\pi - \theta_S} & \theta \geq \theta_S \end{cases} \tag{3}$$

for θ the angle in the anatomical reference system $\{O; V_x, V_y, V_z\}$ and θ_S the angle between V_y and V_x. The septum angular proportion, u_S, is computed as the average of septal angular ranges. The longitudinal parameter v is the angle between a medial axis point and the V_z axis:

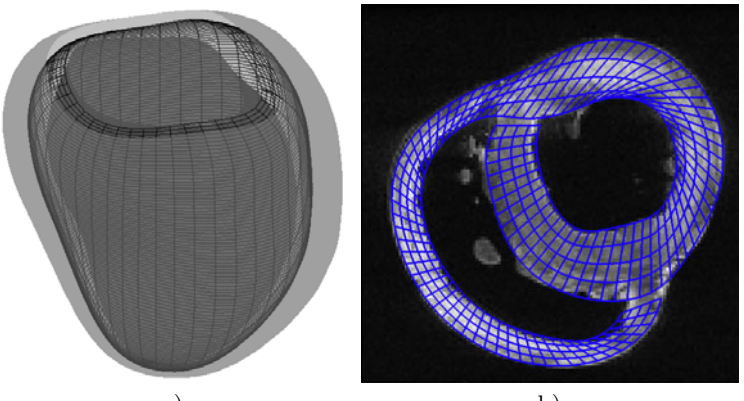
$$v = \pi - \arccos\left(\frac{V_z \cdot \mathbf{x}}{\|\mathbf{x}\|}\right) \tag{4}$$

for $\cdot$ the scalar product. Apical points are assigned $v \equiv 0$ and the basal ring $v \equiv \pi$. Finally, the radial parameter r is defined by enforcing that epicardium is given by $r \equiv -1$, endocardium by $r \equiv 1$ and the medial axis by $r \equiv 0$.

Canonical coordinates have been obtained parametrizing the LV volume with cubic blending functions for the angular parameters and linear for the radial one. The number of considered control points is $17 \times 7 \times 2$. Figure 2 (a) shows canonical geometry inside the LV volume.

4.2 LV-RV Set in 2D SA View

Since short axis views are given by perpendicular planes to LV LA, in this case, the medial axis is a curve, γ, that is homeomorph to a double torus. Inspired



a) b)

Fig. 2. a) Canonical coordinates over the LV volume. b) Canonical coordinates over the LV-RV set.

by dissection methods [11], we use a clock-wise circumferential coordinate, u, starting at V_y and parameterizing first the right ventricle and then the left one. In order to completely unfold the left and right ventricles, the angular coordinate must account for the number of loops, n_l, of the medial axis around the origin O. That is, the angular coordinate is given by the lift of the medial axis path on its universal covering $\mathbb{R}^1$. For each $p \in \gamma$, the lifted angular coordinate, θ_l, is computed by adding $2\pi(nl-1)$ to the angle, θ, measured from V_y to p: $\theta_l = \theta + 2\pi(nl-1)$, for the number of loops n_l given by the number of intersections between the radius through the point p and γ. The circumferential parameter is assigned by mapping the right ventricle and the septal segments to the same angular range $[0, u_S]$ in the parametric domain:

$$u = \begin{cases} \frac{u_S}{\theta_S}\theta + 2\pi(n_l - 1), & \theta \leq \theta_S \\ \\ \frac{2\pi - \theta_S}{2\pi - u_S}\theta + 2\pi\frac{\theta_S - u_S}{2\pi - u_S} & \theta \geq \theta_S \end{cases} \tag{5}$$

for θ_S the angle between V_y and V_x and the angular proportion u_S computed as the average of septal angular ranges [12]. Finally we reverse u in order to follow the medial path from right to left ventricle: $u \rightarrow 2\pi + u_s - u$. Again, the radial parameter, r, is defined by assigning $r \equiv -1$ to epicardium, $r \equiv 1$ to endocardium and $r \equiv 0$ to the medial axis.

Figure 3 shows an example of the parametrization of the myocardium in short axis view using tubular coordinates. The assignment of the circumferential parameter following the dissection path is given in fig. 3(a). The modelling of endocardial (green solid line) and epicardial (red solid line) walls together with the medial axis (black dashed line) is shown in fig.3(b).

Canonical coordinates have been obtained parametrizing the LV-RV set with cubic blending functions for the angular parameter and linear for the radial one. Since the joined geometry of both ventricles is more complex than the left

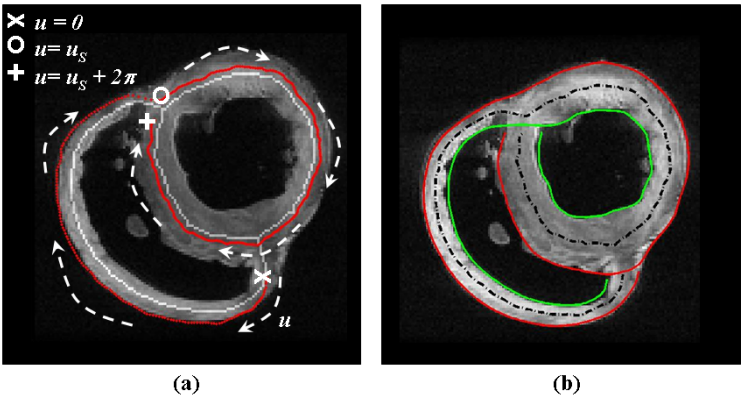


Fig. 3. Parametrization of the joined geometry of the right and left ventricles: circumferential parameter for the medial axis, (a), and tubular parametrization, (b)

ventricle alone, the number of control points has increased to 25×2. Figure 2 (b) shows canonical geometry over the LV-RV set.

5 Statistical Models

In both, the 2D and 3D cases, we use the analytical formulations of the canonical coordinates, given by B-Spline parametrizations and apply PCA to their control points. This provides an statistical model codifying the relation between the object boundaries and its interior.

Regarding LV volume we considered, as training set, $N = 8$ DTI unwaighted volumes belonging to normal canine hearts, freely available at web of The Center for Cardiovascular Bioinformatics and Modeling (`www.ccbm.jhu.edu`). Since the number of instances in the training set is smaller than the space dimension, at most N eigenvectors (spanning the subspace generated by the elements in the training set) can be obtained. In this case we have taken the first 6 modes of variation (which explain a 99.25% of the total shape variability). The variability associated to the first 5 modes is shown in figure 4 (above).

Regarding LV-RV structure we considered, as training set, 45 standard MR SA slices belonging to both, Basal and Mid levels. For the statistical model we have taken the first 5 modes of variation (which explain a 95.6% of the total shape variability). The variability associated to the first 5 modes is shown in figure 4 (below). Each row corresponds to a mode of variation (from left to right) in the range $\pm 2\sqrt{\lambda_n}$ (for $\sqrt{\lambda_n}$ the standard deviation associated to the mode).

6 Final Remarks

We have presented a mathematical framework for endowing canonical geometries to anatomical structures, by means of tubular coordinates. This establishes correspondences between subjects allowing the combined analysis of shape and appearance. This framework facilitates moving over the target structure and is suitable for defining regular meshes, facilitating further simulation or finite differences schemes.

Analytical formulation of the geometry is given using B-Splines. Since they have vector space structure, statistical analysis measuring the variability in the relations between boundaries and their interior, are obtained applying PCA to the control points.

We have applied our methodology to two challenging cardiac imaging applications. On one hand, we have provided a model of the left ventricle including the basal ring, which cannot be easily modelled using regular meshes or diffeomorphic maps. On the other hand, we have approached the geometry of the right and left ventricles, which, by their loop distribution, is a delicate step. Currently we are extending the methodology to the 3D LV-RV set and preliminary results are promising, however the extension to the whole heart (including atria) seems unfeasible so far.

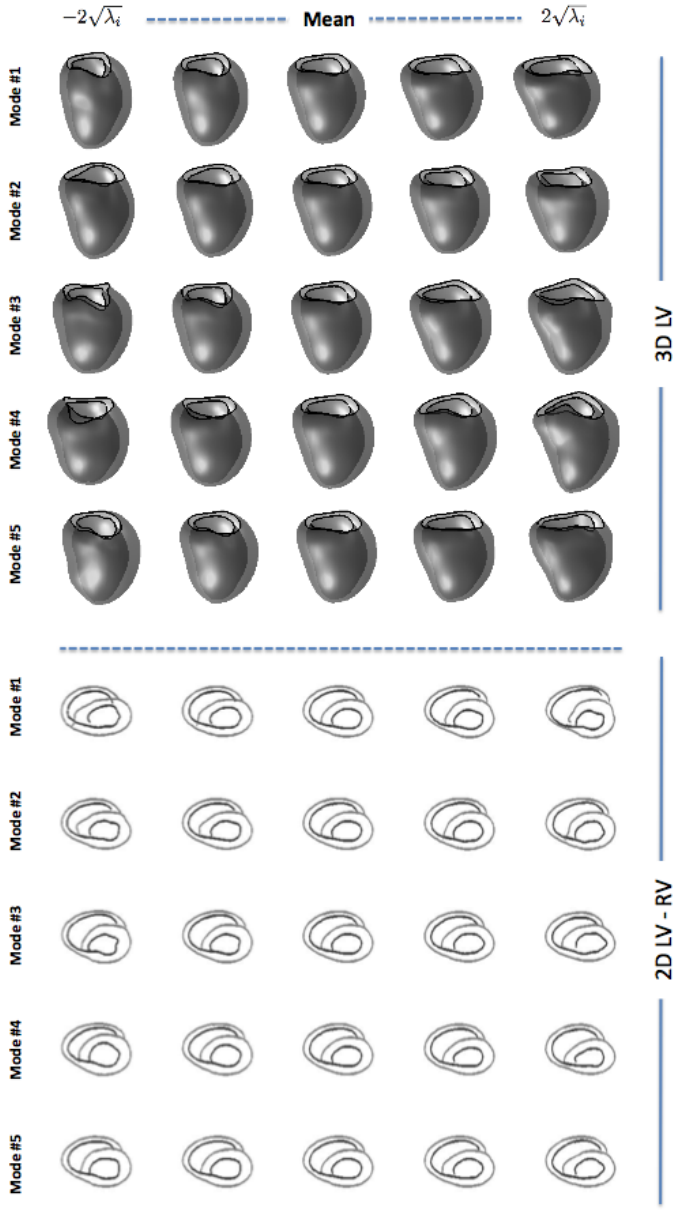


Fig. 4. First 5 Modes of variation performed over the control points of the B-Spline parametric maps that endow a canonical geometry to the LV volume (above) and to the 2D SA LV-RV set (below)

The proposed methodology is easy to implement and computationally is more efficient than other proposed approaches based on PDE. The framework is fed by a ROI enclosing the object under study: the LV in the 3D case and the LV-RV

set in the 2D case. Thus, the speed of the whole process mostly depends on the chosen segmentation method (which is out of the scope of this work). Regarding landmarks, Apical Cap and Basal Ring points can be automatically detected from the 3D ROI, whereas LV-RV junctions require manual intervention. In the 2D case, junction points are directly selected from the MR image and in the 3D case these 2 landmarks are selected over the image obtained after considering the mean along the Z-direction of the unweighted DTI volume.

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